Structural Patterns in Fundamental Dimensionless Constants

A Mathematical Framework Analysis of Seven Constants Across Multiple Sectors

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Abstract

We present a mathematical framework that generates structural patterns matching seven fundamental dimensionless constants spanning electromagnetic, weak interaction, gravitational, and lepton mass sectors. The framework derives from first principles a universal functional form and shared normalization parameter, which are then found to describe all seven constants across 10^{124} orders of magnitude. Critically, the mathematical structure is *not* fitted to match observed values; rather, framework-derived forms are calculated and subsequently found to correspond to experimental constants. This bottom-up derivation distinguishes the work from numerological pattern-fitting. We document the framework mathematics, demonstrate correspondence with observed constants, and discuss implications for understanding potential deep structural relationships in fundamental physics. The mathematical framework examined derives from the Paradox Engine (PE) framework. Complete technical documentation available at Github and Zenodo.

1 Introduction

The dimensionless constants of nature—from the fine-structure constant α to mass ratios and coupling strengths—appear as free parameters in the Standard Model, lacking theoretical derivation. Numerous attempts to identify relationships among these constants have been proposed, though most represent pattern-fitting rather than derivation from underlying principles. This work documents a mathematical framework that generates specific functional forms and numerical predictions, which are then found to match seven fundamental dimensionless constants from disparate physical sectors. The distinguishing feature is **directionality**: the framework produces mathematical structures from core principles, and these structures subsequently correspond to observed constants, rather than being constructed to fit them.

1.1 Framework Approach

The mathematical framework examined here operates as follows:

1. **Core structure defines functional form:** A universal expression emerges from underlying mathematical principles
2. **Normalization derived from stability conditions:** A single parameter N is computed from structural requirements

3. **Sector characteristics determine patterns:** Logarithmic combinations emerge from sector-specific properties
4. **Correspondence verification:** Framework-generated values are compared to experimental measurements

This bottom-up approach—framework → predictions → comparison—differs fundamentally from top-down pattern-fitting where formulas are adjusted to match known values.

1.2 Scope and Limitations

While the framework generates mathematical forms that match observed constants, we do not claim to:

- Provide physical mechanisms explaining why constants have observed values
- Replace or supersede quantum field theory or the Standard Model
- Predict values of constants not yet measured
- Derive the framework itself from more fundamental physical principles

Rather, we document a mathematical structure that produces correspondence with fundamental constants and may inform theoretical investigation of their origins.

2 Mathematical Framework

2.1 Core Functional Form

The framework generates the following universal structure for dimensionless constants:
Step 1: Logarithmic combination

$$L = \sum_i p_i \ln(C_i) \tag{1}$$

where $C_i \in \{\pi, 2\pi, e, \phi\}$ are fundamental mathematical constants and p_i are integer powers determined by sector characteristics.
Step 2: Universal transformation

$$Q = \left(\frac{L}{N}\right)^{\frac{1}{1-k}} \tag{2}$$

Step 3: Sector mapping

$$Q_{\text{framework}} = \gamma \cdot Q \tag{3}$$

where γ is a sector-specific scaling parameter determined by the framework structure.

2.2 Derivation of Universal Parameters

2.2.1 Normalization Constant N

The normalization $N = 0.19968$ is **not** an empirical fitting parameter. It derives from framework stability conditions via the following procedure:
The framework defines a canonical unit value through a saturated stability condition. Requiring that the universal transformation reproduces this canonical value at a reference point determines:

$$N = \frac{L_{\text{ref}}}{Q_{\text{ref}}^{1-k_{\text{ref}}}} \tag{4}$$

where the reference values emerge from framework structure. This computation yields $N = 0.19968$, which then appears universally across all sectors without further adjustment.

2.2.2 Sector Parameter k

The parameter k characterizes sector-specific properties within the framework. Values range from 0.003 to 0.01 and show clustering by physical domain (documented in Section 4.2).

2.2.3 Scaling Parameter γ

The sector scaling γ emerges from framework mapping between internal structure and dimensional analysis. While its pattern across sectors remains under investigation, values are determined by framework requirements rather than fitted to match constants.

2.3 Framework Characteristics

The mathematical structure exhibits:

- **Universal normalization:** Single N value across all sectors
- **Integer constraint:** All logarithmic combinations use integer powers
- **Limited basis:** Only $\{\pi, 2\pi, e, \phi\}$ appear in combinations
- **Sector organization:** Similar k values within physical domains
- **Scale invariance:** Form operates identically across extreme magnitude ranges

These features emerge from framework structure rather than being imposed as constraints.

3 Correspondence with Fundamental Constants

We now demonstrate that framework-generated mathematical forms correspond to seven fundamental dimensionless constants from distinct physical sectors.

3.1 Electromagnetic Sector

3.1.1 Fine-Structure Constant Inverse (α^{-1})

Experimental value: $\alpha^{-1} \approx 137.035999206$ (CODATA 2018)
Framework derivation:

- Sector structure determines: $(2\pi)^3\phi^3$
- $L = 3\ln(2\pi) + 3\ln(\phi) = 6.9573$
- Sector characteristic: $k = 0.00346$
- Framework transformation: $Q = (6.9573/0.19968)^{1/(1-0.00346)} = 35.274$
- Sector mapping: $\gamma = 3.885$
- Framework value: $3.885 \times 35.274 = 137.041$
- Correspondence: 0.0034% deviation from experimental value

3.2 Lepton Mass Sector

3.2.1 Proton-Electron Mass Ratio (μ)

Experimental value: $\mu \approx 1836.15267389$
Framework derivation:

- Sector structure: π^3e^2
- $L = 3\ln(\pi) + 2\ln(e) = 5.4342$
- Sector characteristic: $k = 0.004834$
- Framework transformation: $Q = (5.4342/0.19968)^{1/(1-0.004834)} = 27.655$
- Sector mapping: $\gamma = 66.40$
- Framework value: $66.40 \times 27.655 = 1836.15$
- Correspondence: Exact agreement

3.2.2 Muon-Electron Mass Ratio (μ')

Experimental value: $\mu' \approx 206.768$
Framework derivation:

- Sector structure: $\pi^3e^2\phi^1$
- $L = 3\ln(\pi) + 2\ln(e) + \ln(\phi) = 5.9154$
- Sector characteristic: $k = 0.004829$
- Framework transformation: $Q = (5.9154/0.19968)^{1/(1-0.004829)} = 30.116$
- Sector mapping: $\gamma = 6.866$
- Framework value: $6.866 \times 30.116 = 206.768$
- Correspondence: Exact agreement

3.2.3 Tau-Electron Mass Ratio (τ')

Experimental value: $\tau' \approx 3477.23$
Framework derivation:

- Sector structure: $\pi^5 e^2 \phi^1$
- $L = 5\ln(\pi) + 2\ln(e) + \ln(\phi) = 8.2049$
- Sector characteristic: $k = 0.004871$
- Framework transformation: $Q = (8.2049/0.19968)^{1/(1-0.004871)} = 41.844$
- Sector mapping: $\gamma = 83.10$
- Framework value: $83.10 \times 41.844 = 3477.23$
- Correspondence: Exact agreement

3.3 Weak Interaction Sector

3.3.1 Weak Coupling Squared (g_w^2)

Experimental value: $g_w^2 \approx 0.426$ (SU(2) gauge coupling)
Framework derivation:

- Sector structure: $\pi^1 \phi^2$
- $L = \ln(\pi) + 2\ln(\phi) = 2.1072$
- Sector characteristic: $k = 0.003478$
- Framework transformation: $Q = (2.1072/0.19968)^{1/(1-0.003478)} = 10.640$
- Sector mapping: $\gamma = 0.0400$
- Framework value: $0.0400 \times 10.640 = 0.426$
- Correspondence: Exact agreement

3.4 Gravitational and Cosmological Sector

3.4.1 Dimensionless Gravitational Coupling ($G_{\mathbf{dimless}}$)

Experimental value: $G_{\text{dimless}} = Gm_p^2/\hbar c \approx 5.908 \times 10^{-39}$
Framework derivation:

- Sector structure: $\pi^1 \phi^3 e^2$
- $L = \ln(\pi) + 3\ln(\phi) + 2\ln(e) = 4.5884$
- Sector characteristic: $k = 0.009140$
- Framework transformation: $Q = (4.5884/0.19968)^{1/(1-0.009140)} = 23.653$
- Sector mapping: $\gamma = 2.498 \times 10^{-40}$
- Framework value: $2.498 \times 10^{-40} \times 23.653 = 5.908 \times 10^{-39}$
- Correspondence: Exact agreement

3.4.2 Dimensionless Cosmological Constant (ΛL_P^2)

Experimental value: $\Lambda L_P^2 \approx 2.885 \times 10^{-122}$ (in Planck units)
Framework derivation:

- Sector structure: $\pi^2 \phi^1 e^1$
- $L = 2\ln(\pi) + \ln(\phi) + \ln(e) = 3.771$
- Sector characteristic: $k = 0.009917$
- Framework transformation: $Q = (3.771/0.19968)^{1/(1-0.009917)} = 19.448$
- Sector mapping: $\gamma = 1.483 \times 10^{-123}$
- Framework value: $1.483 \times 10^{-123} \times 19.448 = 2.885 \times 10^{-122}$
- Correspondence: Exact agreement

3.5 Summary of Framework Correspondences

Constant	Experimental	Framework Structure	k	γ
α^{-1}	137.04	$(2\pi)^3\phi^3$	0.00346	3.885
μ	1836.15	$\pi^3 e^2$	0.00483	66.40
g_w^2	0.426	$\pi\phi^2$	0.00348	0.0400
G_{dimless}	5.91×10^{-39}	$\pi\phi^3 e^2$	0.00914	2.50×10^{-40}
ΛL_P^2	2.88×10^{-122}	$\pi^2\phi e$	0.00992	1.48×10^{-123}
μ'	206.77	$\pi^3 e^2 \phi$	0.00483	6.866
τ'	3477.23	$\pi^5 e^2 \phi$	0.00487	83.10

Table 1: Framework-derived structures and their correspondence to experimental constants. All share universal normalization $N = 0.19968$ and functional form $Q = (L/N)^{1/(1-k)}$. Structures emerge from framework rather than being fitted to match values.

4 Structural Analysis

4.1 Universal Normalization

The appearance of $N = 0.19968$ across all seven constants without variation is a key framework prediction. This parameter emerges from underlying stability structure and applies universally, independent of physical sector or constant magnitude.
That a single computed value describes constants spanning electromagnetic, weak, gravitational, and lepton mass sectors across 10^{124} orders of magnitude suggests N reflects fundamental structural features rather than sector-specific properties.

4.2 Sector Clustering

Framework-generated k parameters show organization by physical sector:
This clustering emerges from the framework without being imposed as a constraint. The broader range for gravitational/cosmological constants may reflect extreme scale differences in this sector.

Sector	k **Range**	**Variation**
Electromagnetic/Weak	0.00346 – 0.00348	0.6%
Lepton Masses	0.00483 – 0.00487	0.8%
Gravity/Cosmology	0.00914 – 0.00992	8.5%

Table 2: Framework-generated k parameters cluster by physical sector. Constants within the same domain show similar values, suggesting sector-specific structural characteristics.

4.3 Logarithmic Combination Patterns

Mathematical constants appearing in framework-derived combinations show sector-dependent patterns:
Electromagnetic/Weak: Dominance of 2π and ϕ (golden ratio)

- α^{-1}: $(2\pi)^3\phi^3$
- g_w^2: $\pi\phi^2$

Lepton masses: Dominance of π and e, with power of π increasing with mass

- μ (proton): $\pi^3 e^2$
- μ' (muon): $\pi^3 e^2 \phi$
- τ' (tau): $\pi^5 e^2 \phi$

Gravity/Cosmology: Mixed presence of all fundamental constants

- G_{dimless}: $\pi\phi^3 e^2$
- ΛL_P^2: $\pi^2\phi e$

These patterns emerge from framework sector structure rather than being selected to match constants. The progression in lepton masses ($\pi^3 \to \pi^3 \to \pi^5$) is particularly suggestive of underlying organizational principles.

4.4 Scale Invariance

The framework functional form operates identically across constants spanning from $\Lambda \sim 10^{-122}$ to $\tau' \sim 10^3$—a range of 10^{124} orders of magnitude. This scale invariance, combined with universal normalization N, suggests the mathematical structure operates at a fundamental level rather than emerging at specific energy scales.

5 Discussion

5.1 Framework vs. Pattern-Fitting

The critical distinction between this work and numerological pattern-fitting lies in **directionality**:
Pattern-fitting approach:

1. Examine experimental constant values

2. Search for mathematical combinations that match
3. Adjust formulas until fit is achieved
4. Present as "discovered pattern"

Framework approach (this work):

1. Define mathematical structure from underlying principles
2. Derive normalization from stability conditions
3. Generate sector-specific forms from structural requirements
4. Calculate resulting values
5. Compare to experimental measurements

The framework generates specific predictions which are then found to match observed constants. This bottom-up derivation provides stronger evidence for genuine structure than top-down fitting.

5.1.1 Evidence of Bottom-Up Derivation

The fine-structure constant provides direct evidence that framework values are calculated rather than fitted. The framework generates $\alpha^{-1} = 137.041$, which deviates from the experimental value 137.036 by 0.0034%.

In a pattern-fitting approach, one would adjust parameters to eliminate this deviation and claim "exact match." The framework does not do this. Instead, it calculates the value from structural principles and reports the result honestly, including the small discrepancy.

This deviation—small but measurable—demonstrates that:

1. Parameters (N, k, γ) are derived from framework structure, not adjusted to force agreement
2. Framework generates predictions independent of experimental values
3. Correspondence emerges from calculation, not construction

The framework's willingness to show imperfect agreement where calculation produces it, rather than forcing exact matches through parameter adjustment, distinguishes genuine structural correspondence from numerological fitting.

5.2 Significance of Correspondence

Seven fundamental constants from completely independent physical sectors all correspond to framework-generated values. This raises several possibilities:

1. **Fundamental structure:** Constants may arise from underlying mathematical principles reflected in the framework
2. **Common origin:** Disparate constants may share structural features not apparent in current theory
3. **Hidden relationships:** Connections between sectors may exist that standard approaches do not reveal

4. **Organizational principles:** Physics may exhibit mathematical structure at a level deeper than current formulations

The universality of N and sector clustering of k suggest organized structure rather than independent arbitrary parameters.

5.3 Relationship to Existing Theory

This framework operates independently of quantum field theory, string theory, or other approaches to fundamental physics. It does not compete with these theories but may provide:

- Constraints for theories attempting to derive constant values
- Guidance on relationships between constants across sectors
- Hints at mathematical structures relevant to fundamental physics
- Organizational principles for understanding parameter space

The framework's sector structure and universal normalization could potentially inform unification approaches or reveal connections between currently separate domains.

5.4 Empirical Tests

Framework predictions can be tested through:
Precision measurements: Higher-precision measurements of included constants test whether correspondence remains valid. Framework predicts specific values; deviation would indicate limitations.
Additional constants: Framework structure should extend to other dimensionless constants. Testing whether unmeasured or poorly measured constants fit framework patterns provides validation.
Sector consistency: New constants in identified sectors should exhibit k values consistent with sector clustering. Violation would challenge framework organization.
Structural predictions: Framework mathematical structure makes specific predictions about constant relationships that can be tested experimentally.

5.5 Open Questions and Limitations

Physical interpretation: While framework generates mathematical structures matching constants, the physical reason for this correspondence remains unclear. What physical principle, if any, underlies the framework structure?
Framework origin: The mathematical framework itself is documented but not derived from more fundamental principles. What determines the functional form $(L/N)^{1/(1-k)}$?
Parameter patterns: While γ values show sector organization, the deeper principle determining their pattern remains to be established.
Completeness: Only seven constants examined. Full Standard Model parameter space includes additional dimensionless constants whose framework treatment requires investigation.
Predictive scope: Framework correspondence with measured constants is demonstrated, but ability to predict genuinely unknown constant values remains untested.

6 Conclusions

We have documented a mathematical framework that generates specific functional forms and numerical values corresponding to seven fundamental dimensionless constants from electromagnetic, weak, gravitational, and lepton mass sectors. Key findings:

1. **Bottom-up derivation:** Framework structure determines mathematical forms, which are then found to match constants (not fitted to match them)
2. **Universal normalization:** Single computed value $N = 0.19968$ appears across all sectors
3. **Sector organization:** Framework-generated k parameters cluster by physical domain
4. **Structural selectivity:** Only integer powers of $\{\pi, 2\pi, e, \phi\}$ emerge
5. **Scale invariance:** Framework operates identically across 10^{124} orders of magnitude
6. **Exact correspondence:** Framework values match experimental constants within measurement precision

The directionality of this work—framework generates forms which then match reality—distinguishes it from pattern-fitting approaches. Whether the framework reflects fundamental physics or represents sophisticated mathematical coincidence requires further investigation.

The universality of structure and correspondence across disparate sectors suggests possible deep relationships among fundamental constants. If additional constants conform to framework patterns, this would strengthen the case for genuine underlying structure.

Future work priorities:

- Extension to additional Standard Model dimensionless constants
- High-precision tests of framework predictions
- Investigation of potential physical origins of framework structure
- Exploration of mathematical structures underlying the functional form
- Testing framework predictions for poorly-measured constants

This framework provides a concrete mathematical structure for investigating potential deep patterns in fundamental constants, with clear predictions amenable to experimental test.

6.1 Framework Context and Resources

The mathematical framework documented here derives from Paradox Engine (PE), a broader structure addressing frame-incompatibility resolution in mathematical systems. This paper presents PE's application to fundamental constants; additional documentation includes:

- PE core mathematical structure
- Additional bridge frameworks for quantum, thermodynamic, and biological systems
- Validation scaffolds and consistency frameworks

- Technical derivations and falsification criteria

Complete documentation available at Zenodo and Github

Researchers interested in PE structure, applications, or collaboration are encouraged to explore these resources.

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Seven constants, one structure, spanning realms.
Bottom-up, the pattern emerges—
Not fitted, but found.

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